Prodotti notevoli

indice

Gli esercizi sono proposti in ordine di difficoltà crescente.

nota: in un file così lungo e complesso può accadere che sia presente un errore di diversa natura nonostante gli esercizi siano stati controllati più volte. Saremo grati di ricevere segnalazioni di eventuali refusi o suggerimenti di qualsiasi natura.

Prodotti notevoli

PASSED

1. somma per differenza

1	$(a-1)(a+1)$	a^2-1
2	$(x+3)(x-3)$	x^2-9
3	$(a-5)(a+5)$	a^2-25
4	$(2x-1)(2x+1)$	$4x^2-1$
5	$(-10y+1)(10y+1)$	$-100y^2+1$
6	$(xy-6)(xy+6)$	x^2y^2-36
7	$(3a-2b)(3a+2b)$	$9a^2-4b^2$
8	$(x^2-3y)(x^2+3y)$	x^4-9y^2
9	$(4x^2-7y)(4x^2+7y)$	$16x^4-49y^2$
10	$(3abc^2+3)(3abc^2-3)$	$9a^2b^2c^4-9$
11	$\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)$	$x^2-\frac{1}{4}$
12	$\left(\frac{11}{9}a-1\right)\left(\frac{11}{9}a+1\right)$	$\frac{121}{81}a^2-1$
13	$\left(+\frac{3}{4}a+b\right)\left(-\frac{3}{4}a+b\right)$	$-\frac{9}{16}a^2+b^2$

14	$\left(\frac{4}{3}ab^2-5c^3\right)\left(\frac{4}{3}ab^2+5c^3\right)$	$\frac{16}{9}a^2b^4-25c^6$
15	$\left(\frac{1}{2}x+\frac{7}{3}y^2\right)\left(\frac{1}{2}x-\frac{7}{3}y^2\right)$	$\frac{1}{4}x^2-\frac{49}{9}y^4$
16	$\left(\frac{2}{3}m-\frac{4}{5}n^2\right)\left(\frac{2}{3}m+\frac{4}{5}n^2\right)$	$\frac{4}{9}m^2-\frac{16}{25}n^4$
17	$\left(3xy^2+\frac{2}{5}x^2\right)\left(3xy^2-\frac{2}{5}x^2\right)$	$9x^2y^4-\frac{4}{25}x^4$
18	$\left(1-\frac{3}{4}a^2b\right)\left(1+\frac{3}{4}a^2b\right)$	$1-\frac{9}{16}a^4b^2$
19	$\left(x^2y^3-\frac{3}{8}m^2n\right)\left(x^2y^3+\frac{3}{8}m^2n\right)$	$x^4y^6-\frac{9}{64}m^4n^2$
20	$(x+2y)(x-2y)(x^2+4y^2)$	x^4-16y^4
21	$(a+3b)(a-3b)(a^2+9b^2)$	a^4-81b^4

2. quadrato di binomio

22	$(a+1)^2$	a^2+2a+1
23	$(x-1)^2$	x^2-2x+1
24	$(a+2)^2$	a^2+4a+4
25	$(x-3)^2$	x^2-6x+9
26	$(4a+1)^2$	$16a^2+8a+1$

27	$(-7b+1)^2$	$1-14b+49b^2$
28	$(a-5b)^2$	$a^2-10ab+25b^2$
29	$(3-7y)^2$	$9-42y+49y^2$
30	$(xy+8)^2$	$x^2y^2+16xy+64$
31	$(5a+2b)^2$	$25a^2+20ab+4b^2$
32	$(3xy^2-x^2)^2$	$9x^2y^4-6x^3y^2+x^4$
33	$\left(x+\frac{1}{3}\right)^2$	$x^2+\frac{2}{3}x+\frac{1}{9}$
34	$\left(\frac{1}{6}x+1\right)^2$	$\frac{1}{36}x^2+\frac{1}{3}x+1$
35	$\left(\frac{1}{5}a+2b\right)^2$	$\frac{1}{25}a^2+\frac{4}{5}ab+4b^2$
36	$\left(2a^2b-\frac{1}{2}ab^2\right)^2$	$4a^4b^2-2a^3b^3+\frac{1}{4}a^2b^4$
37	$\left(-\frac{2}{3}m^2-\frac{1}{2}mn^2\right)^2$	$\frac{4}{9}m^4+\frac{2}{3}m^3n^2+\frac{1}{4}m^2n^4$
38	$\left(-a^2-\frac{7}{4}b^2\right)^2$	$a^4+\frac{7}{2}a^2b^2+\frac{49}{16}b^4$
39	$(-ax^2+3x)^2$	$a^2x^4-6ax^3+9x^2$

40	$(7x^2yz - y^2z)^2$	$49x^4y^2z^2 - 14x^2y^3z^2 + y^4z^2$
41	$\left(\frac{3}{4}a^2b + \frac{1}{4}b^4c\right)^2$	$\frac{9}{16}a^4b^2 + \frac{3}{8}a^2b^{\,5}c + \frac{1}{16}b^8c^2$
42	$(-x^m + 3y^n)^2$	$x^{2m} - 6x^m y^n + 9y^{2n}$
43	$(2a^{3x+1} - 3b^{y-1})^2$	$4a^{6x+2} - 12a^{3x+1}b^{y-1} + 9b^{2y-2}$

3. cubo di binomio

44	$(x + 1)^3$	$x^3 + 3x^2 + 3x + 1$
45	$(a - 1)^3$	$a^3 + 3a^2 + 3a + 1$
46	$(a + 4)^3$	$a^3 + 12a^2 + 48a + 64$
47	$(x - 2)^3$	$x^3 - 6x^2 + 12x - 8$
48	$(1 - 3b)^3$	$1 - 9b + 27b^2 - 27b^3$
49	$(2a + b^2)^3$	$8a^3 + 12a^2b^2 + 6ab^4 + b^6$
50	$(3x - 2)^3$	$27x^3 - 54x^2 + 36x - 8$
51	$(x - 2y)^3$	$x^3 - 6x^2y + 12xy^2 - 8y^3$

52	$\left(-\frac{2}{3}x^2 - xy\right)^3$	$-\frac{8}{27}x^6 - \frac{4}{3}x^5y - 2x^4y^2 - x^3y^3$
53	$\left(\frac{1}{9}a^2b - \frac{1}{2}x^2\right)^3$	$\frac{1}{729}a^6b^3 - \frac{1}{54}a^4b^2x^2 + \frac{1}{12}a^2bx^4 - \frac{1}{8}x^6$
54	$\left(1 - \frac{2}{3}a^2bc\right)^3$	$1 - 2a^2bc + \frac{4}{3}a^4b^2c^2 - \frac{8}{27}a^6b^3c^3$
55	$\left(\frac{1}{2}ab^3 + \frac{4}{3}b^2c\right)^3$	$\frac{1}{8}a^3b^9 + a^2b^8c + \frac{8}{3}ab^7c^2 + \frac{64}{27}b^6c^3$
56	$\left(\frac{1}{3}mn^2 - 3ab^2\right)^3$	$\frac{1}{27}m^3n^6 - ab^2m^2n^4 + 9a^2b^4mn^2 - 27a^3b^6$
57	$\left(\frac{1}{4}ax^2 - \frac{2}{3}xb^2y\right)^3$	$\frac{1}{64}a^3x^6 - \frac{1}{8}a^2b^2x^5y + \frac{1}{3}ab^4x^4y^2 - \frac{8}{27}b^6x^3y^3$
58	$(a^m - 2b^n)^3$	$a^{3m} - 6a^{2m}b^n + 12a^mb^{2n} - 8b^{3n}$

4. potenza di binomio

59	$(a+1)^4$	$a^4 + 4a^3 + 6a^2 + 4a + 1$
60	$(x-1)^4$	$x^4 - 4x^3 + 6x^2 - 4x + 1$
61	$(x+2y)^4$	$x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$
62	$(2x+y)^5$	$32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$
63	$(2a-3b)^4$	$16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$

64	$(1-2x^2)^6$	$1-12x^2+60x^4-160x^6+240x^8-192x^{10}+64x^{12}$
65	$(3a+b^2)^5$	$243a^5+405a^4b^2+270a^3b^4+90a^2b^6+15ab^8+b^{10}$
66	$(-x^2-2y)^6$	$x^{12}+12x^{10}y+60x^8y^2+160x^6y^3+240x^4y^4+192x^2y^5+64y^6$
67	$(-a^3+3)^4$	$a^{12}-12a^9+54a^6-108a^3+81$
68	$(ab+2c)^4$	$a^4b^4+8a^3b^3c+24a^2b^2c^2+32abc^3+16c^4$
69	$(a-2bc)^6$	$a^6-12a^5bc+60a^4b^2c^2-160a^3b^3c^3+240a^2b^4c^4+-192ab^5c^5++64b^6c^6$
70	$\left(\frac{1}{2}x+y^2\right)^5$	$\frac{1}{32}x^5+\frac{5}{16}x^4y^2+\frac{5}{4}x^3y^4+\frac{5}{2}x^2y^6+\frac{5}{2}xy^8+y^{10}$

5. quadrato di trinomio

71	$(x+y+1)^2$	$x^2+y^2+1+2xy+2x+2y$
72	$(x+y-a)^2$	$x^2+y^2+a^2+2xy-2xa-2ya$
73	$(a-x-1)^2$	$a^2+x^2+1-2ax-2a+2x$
74	$(a-b+c)^2$	$a^2+b^2+c^2-2ab-2ac-2ac$
75	$(a+b-2c)^2$	$a^2+b^2+4c^2+2ab-4ac-4bc$

76	$(2a-b-3c)^2$	$4a^2+b^2+9c^2-4ab-12ac+6bc$
77	$(x^2-2x+1)^2$	$x^4-4x^3+6x^2-4x+1$
78	$\left(\frac{1}{2}m-2n+3q\right)^2$	$\frac{1}{4}m^2+4n^2+9q^2-2mn+3mq-12nq$
79	$\left(\frac{2}{3}a^2-3a+2\right)^2$	$\frac{4}{9}a^4-4a^3+\frac{35}{3}a^2-12a+4$
80	$(a^2-bc+2c)^2$	$a^4+b^2c^2+4c^2-2a^2bc+4a^2c-4bc^2$
81	$\left(2x-3y-\frac{1}{4}\right)^2$	$4x^2+9y^2+\frac{1}{16}-12xy-x+\frac{3}{2}y$
82	$(4a^2+3b^2-5c)^2$	$16a^4+9b^4+25c^2+24a^2b^2-40a^2c-30b^2c$
83	$\left(-\frac{2}{3}x^2-3xy-z\right)^2$	$\frac{4}{9}x^4+9x^2y^2+z^2+4x^3y+\frac{4}{3}x^2z+6xyz$
84	$(a^m+b^n-b^p)^2$	$a^{2m}+b^{2n}+b^{2p}+2a^mb^n-2a^mb^p-2b^{n+p}$

6. particolari prodotti notevoli

85	$(a+b+c)(a+b-c)$	$a^2+2ab+b^2-c^2$
86	$(a-b+c)(a+b-c)$	$a^2-b^2+2bc-c^2$
87	$(x+y+1)(-x+y-1)$	y^2-x^2-2x-1

88	$(2x-y+z)(2x+y+z)$	$4x^2+z^2+4xz-y^2$
89	$(x-2y+2)(x-2y-2)$	$x^2-4xy+4y^2-4$
90	$(3x^2-2xy+2y^2)(3x^2+2xy+2y^2)$	$9x^4+8x^2y^2+4y^4$
91	$(2x^2-2xy+y^2)(2x^2+2xy+y^2)$	$4x^4+y^4$
92	$(2a^2+9b^2-6ab)(2a^2+9b^2+6ab)$	$4a^4+81b^4$
93	$(3x^2-1-2x)(-3x^2+1-2x)$	$-9x^4+10^2-1$
94	$(3a-b+2)(3a+b-2)$	$9a^2-b^2-4+4b$
95	$(a^2+ab+b^2)(a^2-ab-b^2)$	$a^4-a^2b^2-b^4-2ab^3$
96	$(-3a-x^2-y)(x^2+y-3a)$	$9a^2-x^4-y^2-2x^2y$
97	$\left(3a-\frac{1}{2}b-2c\right)\left(3a+\frac{1}{2}b+2c\right)$	$9a^2-\frac{1}{4}b^2-2bc-4c^2$
98	$\left(\frac{1}{3}a-b+2\right)\left(\frac{1}{3}a-2-b\right)$	$-2x^2y^2-\frac{1}{4}x^4-9y^4$
99	$\left(-\frac{1}{2}x^2-xy+3y^2\right)\left(\frac{1}{2}x^2-xy-3y^2\right)$	$4x^2y^2-\frac{1}{4}x^4-9y^4$
100	$\left(2x+\frac{5}{3}xy+\frac{3}{2}y\right)\left(\frac{5}{3}xy+\frac{3}{2}y-2x\right)$	$\frac{25}{9}x^2y^2+\frac{9}{4}y^2+5xy^2-4x^2$

101	$\left(\frac{3}{2}a-\frac{1}{2}b-1\right)\left(\frac{3}{2}a+\frac{1}{2}b+1\right)$	$\frac{9}{4}a^2-\frac{1}{4}b^2-b-1$
102	$(a+b+x-y)(a+b-x+y)$	$a^2+2ab+b^2-x^2+2xy-y^2$
103	$(2a^2-ab-3b-a^2b)(2a^2-ab+3b+a^2b)$	$4a^4-4a^3b-5a^2b^2-9b^2-a^4b^2$
104	$(2bx+3y-3x-2by)(2bx-3y+3x-2by)$	$4b^2x^2-8b^2xy+4b^2y^2-9x^2-9y^2+18xy$
105	$(3a-2b+2c-4d)(3a-2b-2c+4d)$	$9a^2-12ab+4b^2-4c^2+16cd-16d^2$
106	$(5x-6ay+2ax-3y)(5x-6ay-2ax+3y)$	$25x^2-48axy+36a^2y^2-4a^2x^2-9y^2$
107	$\left(\frac{1}{2}a-\frac{2}{3}bx-\frac{1}{2}x+\frac{2}{3}ab\right)\left(\frac{1}{2}a-\frac{2}{3}bx+\frac{1}{2}x-\frac{2}{3}ab\right)$	$\frac{1}{4}a^2+\frac{4}{9}b^2x^2-\frac{1}{4}x^2-\frac{4}{9}a^2b^2$

7. esercizi di riepilogo

108	$\left(\frac{1}{2}a-b^2+2c\right)\left(\frac{1}{2}a+b^2+2c\right)$	$\frac{1}{4}a^2+2ac+4c^2-b^4$
109	$\left(-\frac{9}{2}xy-\frac{1}{4}x^2y^2\right)^2$	$\frac{81}{4}x^2y^2+\frac{9}{4}x^3y^3+\frac{1}{16}x^4y^4$
110	$\left(\frac{3}{7}a^2b-\frac{7}{2}b^2c^2\right)^2$	$\frac{9}{49}a^4b^2-3a^2b^3c^2+\frac{49}{4}b^4c^4$
111	$\left(\frac{3}{2}x^2y-\frac{2}{3}xy^2\right)^3$	$\frac{27}{8}x^6y^3-\frac{9}{2}x^5y^4+2x^4y^5-\frac{8}{27}x^3y^6$
112	$\left(\frac{7}{2}ab^2x-\frac{1}{3}xb\right)^3$	$\frac{343}{8}a^3b^6x^3-\frac{49}{4}a^2b^5x^3+\frac{7}{6}ab^4x^3-\frac{1}{27}b^3x^3$
113	$(3m-2n^2)^6$	$729m^6-2916m^5n^2+4860m^4n^4-4320m^3n^6+2160m^2n^8-576mn^{10}+64n^{12}$

114	$\left(\frac{1}{2}x+y^2\right)^4$	$\frac{1}{16}x^4+\frac{1}{2}x^3y^2+\frac{3}{2}x^2y^4+2xy^6+y^8$
115	$(2x^2y-xy^2-x)^2$	$4x^4y^2+x^2y^4+x^2-4x^3y^3-4x^3y+2x^2y^2$
116	$\left(\frac{2}{5}x-y^2+\frac{3}{2}xy\right)^2$	$\frac{4}{25}x^2+y^4+\frac{9}{4}x^2y^2-\frac{4}{5}xy^2+\frac{6}{5}x^2y-3xy^3$

8. espressioni con prodotti notevoli

117	$(x+1)(x-1)+(x+1)^2$	$2x^2+2x$
118	$(x+1)^2-(x+1)(x-1)$	$2x+2$
119	$(a+1)^2-(a+1)^3$	$-a^3-a^2-a$
120	$(a+1)(a-1)(a^2+1)-(a^2+1)^2$	$-2a^2-2$
121	$(a+b)^2-2b(a-b)-(a+b)(b-a)$	$2a^2+2b^2$
122	$4(a-b)+(a-b)^2-(2+a-b)^2$	-4
123	$(2a-1)^2-(a+2)^2+(a+4)^2$	$4a^2+13$
124	$\left(a+\frac{1}{3}x\right)^2-\left(a-\frac{1}{2}x\right)\left(a+\frac{1}{2}x\right)+\frac{11}{36}x^2$	$\frac{2}{3}ax+\frac{2}{3}x^2$
125	$(2x-3)^3+2x(2x-5)^2-4(2x-1)^2(x-3)+15$	$52x-12x^2$
126	$\left(\frac{1}{3}x-y\right)^2\left(\frac{1}{3}x+y\right)-\left(\frac{1}{3}x+y\right)^3$	$-\frac{4}{9}x^2y-\frac{4}{3}xy^2$
127	$\frac{1}{3}(x+b)^2-\frac{4}{9}b^2+\left(\frac{1}{6}x+\frac{1}{3}b\right)\left(\frac{1}{3}b-\frac{1}{6}x\right)$	$\frac{2}{3}bx+\frac{11}{36}x^2$

128	$(x+y)^2(x-y)^2-x^3(x+2y)+[x^2-(x-y)^2](x+y)^2$	x^2y^2
129	$[(2-ab)^2-(ab-1)(ab+1)](5+4ab)-(5-4ab)(5+4ab)$	0
130	$7(4+a)(a-4)+2(a-2)(a-3)+5(a+1)^2$	$14a^2-95$
131	$(2x^2+y)^2-(2x^2+y)(2x^2-y)-4y(x)^2$	$2y^2$
132	$(2x^2+y)(2x^2-y)+(2x^2+y)^2-(2x^2-y)^2+4x^2y:\left(-\frac{1}{2}\right)$	$4x^4-y^2$
133	$(a^2-3b^2)^2+\left(2a^2-\frac{1}{3}b^2\right)\left(2a^2+\frac{1}{3}b^2\right)-\left(\frac{4}{3}a^2-3b^2\right)^2$	$\frac{29}{9}a^4-\frac{1}{9}b^4+2a^2b^2$
134	$(x^3+y^3)(x^3-y^3)+(x+y)^3(x-y)^3-3x^2y^2(x+y)(y-x)+8y^7:4y$	$2x^6$
135	$a^2(a-3)(a+2)+(a^2-3a+2)^2-2a(a-1)^3+a(a^2+10)$	a^2+4
136	$(a+b-2)^2-(a+b+2)^2-4(a+b+2)+12(a+b)$	-8
137	$\left(a+\frac{1}{3}b\right)\left(\frac{1}{3}b-a\right)-\left(b-\frac{1}{2}a\right)^2-ab$	$-\frac{5}{4}a^2-\frac{8}{9}b^2$
138	$(1+a^2)^3-(1+a^3)^2-3a^2(1+a)(1-a)-2a^3(3a+1)$	$-4a^3$
139	$(x-y+z)(x-y-z)-(x+y+z)(x-y-z)+2y(x-y)$	2xy
140	$(x^2-x^4)(x^2+x^4)-(x^2-1)^2+[(x+1)(x-1)(x^2+1)+2]^2$	$2x^4+2x^2$
141	$\left(\frac{2}{5}a-b^2\right)\left(b^2+\frac{2}{5}a\right)-\left[\left(\frac{1}{2}a+\frac{2}{3}b\right)^3-\frac{8}{27}b^3-\frac{1}{6}ab(3a+4b)\right]:\frac{25}{32}a$	$-b^4$

142	$\left(\frac{4}{3}ab+3\right)\left(3-\frac{4}{3}ab\right)+a\left(\frac{4}{3}a+\frac{2}{3}b^2\right)^2-(4a+6)\left(\frac{4}{9}a^2+1-\frac{2}{3}a\right)+\frac{5}{9}ab^4$	$3+ab^4$
143	$[-9xy(3y^2+1)+(-x^2y-1)^2-(3x+9xy^2)(-3y)-(1+x^2y)^2]:x^2$	0