Punti di massimo e minimo relativi ed assoluti

indice

Gli esercizi sono proposti in ordine di difficoltà crescente.

nota: in un file così lungo e complesso può accadere che sia presente un errore di diversa natura nonostante gli esercizi siano stati controllati più volte. Saremo grati di ricevere segnalazioni di eventuali refusi o suggerimenti di qualsiasi natura.

Punti di massimo e minimo relativi ed assoluti

PASSED

1. determinare i punti di massimo e minimo relativo delle seguenti funzioni

1	$y = 3x^2 - 7x + 8$	$\frac{7}{6}$ min
2	$y = -x^2 - 5x + 3$	$-\frac{5}{2}$ max
3	$y = -x^2 + 4x - 3$	2 max
4	$y = x^3 + x^2 + x$	*nessun massimo o minimo*
5	$y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$	1 max 2 min
6	$y = \frac{1}{5}x^5 - 3x^4$	0 max 12 min
7	$y = -x^3 + 2x^2 + x - 3$	$\frac{2}{3} - \frac{\sqrt{7}}{3}$ min $\frac{2}{3} + \frac{\sqrt{7}}{3}$ max
8	$y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x - 1$	-1 max 2 min
9	$y = \frac{x^2 + 2x - 1}{x - 2}$	$2 - \sqrt{7}$ max $2 + \sqrt{7}$ min

10	$y = \frac{x+1}{x+3}$	*nessun massimo o minimo*
11	$y = \frac{x^2-3}{x+2}$	-3 *max* -1 *min*
12	$y = \frac{x^2}{3x^2-x+3}$	0 *min* $6+$ *max*
13	$y = \frac{2x^2}{\sqrt{x^4-8x}}$	*nessun massimo o minimo*
14	$y = \frac{x+4}{\sqrt{x^2+4}}$	1 *max*
15	$y = \sqrt{x^2+3x+5}$	$-\frac{3}{2}$ *min*
16	$y = \frac{\sqrt{x^2-9}}{x^2+3}$	$\pm\sqrt{21}$ *max*
17	$y = \frac{x+4}{\sqrt{x^2-9}}$	*nessun massimo o minimo*
18	$y = \frac{1}{x^2-3x+2}$	$\frac{3}{2}$ *max*

19	$y = 3x^3 - 27x^2 + 1$	$0 \quad max$ $6 \quad min$
20	$y = x^3 - 3x + 7$	$-1 \quad max$ $1 \quad min$
21	$y = x^3 - 9x^2 + 15x + 3$	$1 \quad max$ $5 \quad min$
22	$y = (x-2)^3 + 3(x-2)^2 - 3$	$0 \quad max$ $2 \quad min$
23	$y = \frac{x^2 - 3}{x + 1}$	$non\ ammette\ min\ e\ max$
24	$y = \frac{1}{x^4 + 2x^3}$	$-\frac{3}{2} \quad max$
25	$y = \frac{1}{4(x-2)} - \frac{1}{4(x+2)}$	$0 \quad max$
26	$y = \frac{1}{x^2 + x + 7}$	$-\frac{1}{2} \quad max$
27	$y = 1 - \frac{2}{(x-1)\,(x+1)}$	$0 \quad max$

28	$y = \frac{1}{x^2 - 3x + 2}$	$\frac{3}{2}$ max
29	$y = \frac{x - 1}{x^2 - 5x + 6}$	$1 - \sqrt{2}$ min $1 + \sqrt{2}$ max
30	$y = \frac{4 - x^2}{2x + 5}$	-4 min -1 max
31	$y = \frac{1 + 3x}{x - x^2}$	-1 max $\frac{1}{3}$ min
32	$y = \frac{2x - 1}{(x - 1)^2}$	0 min
33	$y = \frac{x^2 - 4x - 3}{x + 2}$	-5 max 1 min
34	$y = \frac{x^2 - 2x - 2}{x + 1}$	-2 max 0 min
35	$y = \frac{3 - 2x - x^2}{x - 2}$	$2 - \sqrt{5}$ min $2 + \sqrt{5}$ max
36	$y = \frac{6x^2 - x - 1}{x^2 + 1}$	$-7 - 5\sqrt{2}$ max $-7 + 5\sqrt{2}$ min

37	$y = \frac{x^2+3}{x-5}$	$5-2\sqrt{7}$ *max* $5+2\sqrt{7}$ *min*
38	$y = x(1-3x)^4$	$\frac{1}{15}$ *max* $\frac{1}{3}$ *min*
39	$y = \frac{x^2+4x}{x^2+6x+5}$	*non ammette min e max*
40	$y = x^3\sqrt{x}$	0 *min*
41	$y = x + x^{\frac{2}{3}}$	0 *min*
42	$y = x^2 e^{-x+1}$	0 *min* 2 *max*
43	$y = e^{x^2-2x}(x^2-2x+3)$	1 *min*
44	$y = xe^{\frac{1}{x}}$	1 *min*
45	$y = ln(x^2-x+1)$	$\frac{1}{2}$ *min*

46	$y = x^{-2}e^{x}$	$2 \quad min$
47	$y = e^{\frac{4x^2}{2x-2}}$	$0 \quad max$ $2 \quad min$
48	$y = 8lnx - 32x$	$\frac{1}{4} \quad max$
49	$y = \frac{x^2 - x - 4}{x - 1}$	$non\ ammette\ min\ e\ max$
50	$y = \sqrt{x^2 + 2x}$	$-2 \quad min$ $0 \quad min$
51	$y = \frac{3x^2}{\sqrt{2x - 1}}$	$\frac{2}{3} \quad min$
52	$y = \frac{x}{(x^2 + 1)\sqrt{x^2 + 1}}$	$-\frac{\sqrt{2}}{2} \quad min$ $\frac{\sqrt{2}}{2} \quad max$
53	$y = \frac{x^2 - 1}{ln(x^2 - 1)}$	$\pm\sqrt{e + 1} \quad min$

54	$y = ln\sqrt{x^2 - 5x + 6}$	*nessun massimo o minimo*
55	$y = lnx^2 - x$	$2 \quad max$
56	$y = \frac{1}{2e^{x^2}}$	$0 \quad max$
57	$y = x^2 lnx$	$\frac{1}{\sqrt{e}} \quad min$
58	$y = (2x + 1)e^{-3x}$	$-\frac{1}{6} \quad max$
59	$y = \frac{e^{2x}}{3x + 1}$	$\frac{1}{6} \quad min$
60	$y = e^{x^5 - 6x^2}$	$0 \quad max$ $\sqrt[3]{\frac{12}{5}} \quad min$
61	$y = e^{\frac{x^2}{1-x}}$	$0 \quad min$ $2 \quad max$
62	$y = ln\frac{x^2 - 4}{x^2 - 1}$	$0 \quad min$

63	$y = \frac{lnx}{x}$	$e \quad max$
64	$y = \frac{1}{ln(sin\,x + 1)}$	$\frac{\pi}{2} + 2k\pi \quad min$
65	$y = -\sqrt{3}\,sin\,x + cos\,x + 6$	$\frac{2}{3}\pi + 2k\pi \quad min$ $\frac{5}{3}\pi + 2k\pi \quad max$
66	$y = (1 - cos\,x)\,sin\,x$	$\frac{2}{3}\pi + 2k\pi \quad max$ $\frac{4}{3}\pi + 2k\pi \quad min$
67	$y = \frac{sin\,x + 1}{cos\,x + 1}$	$\frac{3}{2}\pi + 2k\pi \quad min$
68	$y = \frac{sin\,x - 1}{cosx - 1}$	$\frac{\pi}{2} + 2k\pi \quad min$
69	$y = 2\sqrt{2}\,sin\,x + 2cosx$	$arctan\sqrt{2} + k\pi \quad max$
70	$y = tan\,x + cotan\,x - \frac{1}{2}$	$\frac{\pi}{4} + k\pi \quad min$ $\frac{3}{4}\pi + k\pi \quad max$

71	$y = \cos x + \sin x + \cos x$	$in \quad [0; 2\pi]$	$\frac{1}{\tan\frac{1}{2}} \quad max$ $\frac{5}{6}\pi \quad min$
72	$y = \cos x \ \sqrt[3]{\sin x}$	$in \quad \left[0; \frac{\pi}{2}\right]$	$\frac{\pi}{6} \quad max$
73	$y = 2\cos^2\left(\frac{x}{2}\right)$	$in \quad \left[\frac{\pi}{2}; \pi\right]$	$\pi \quad min$ $2\pi \quad max$
74	$y = \frac{\sin x}{1 + 2\sin^2 x}$	$in \quad [0; \pi]$	$\frac{\pi}{4} \quad max$ $\frac{\pi}{2} \quad min$
75	$y = ln\sqrt{x^2 + 1} + \tan^{-1} x$		$-1 \quad min$
76	$y = \frac{x^3}{3} - \sin x + \cos x$		*non ammette min e max*
77	$y = \frac{1}{\tan^2 x} + \frac{2}{\frac{\sin x}{\cos x}}$	$in \quad [0; \pi]$	$\frac{3}{4}\pi \quad max$
78	$y = \tan^{-1} 2x + ln\,(1 + 4x)^{\frac{1}{2}}$		*non ammette min e max*

79	$y = 12x + 6\,sin^{-1}\,x$	*non ammette min e max*
80	$y = \dfrac{12}{4\,sin\,x}$ $[0; 2\pi]$	$\frac{\pi}{2}$ *min* $\frac{3}{2}\pi$ *max*
81	$y = \lvert x(x-4)\rvert$	0 *min* 2 *max* 4 *min*
82	$y = \left\lvert\dfrac{x-1}{x+2}\right\rvert$	1 *min*
83	$y = \begin{cases}(x+1)(x^2-x+1) & per\ x \le 0\\ x^4-4x+1 & per\ x > 0\end{cases}$	0 *max* 1 *min*
84	$y = \begin{cases}\dfrac{9}{x+2} & per\ x < 1\\ x^{\frac{1}{2}}+2 & per\ x \ge 1\end{cases}$	*non ammette min e max*
85	$y = \begin{cases}(5-x^2)^{\frac{1}{2}} & per\ -2 \le x < 1\\ \dfrac{16}{10-2x} & per\ x \ge 1\end{cases}$	0 *max*

2. determinare i punti di massimo e minimo assoluto delle seguenti funzioni nell'intervallo indicato

86	$y = 2x^2 + 4x - 3$	$-3 \le x \le 5$	-1 *min* 5 *max*

87	$y = x^4 + x^3 + 2x^2 - 11x$	$0 \le x \le e$	$1 \quad max$ $e \quad min$
88	$y = x^3 - \frac{5}{2}x^2 + 2x + \frac{3}{4}$	$-1 \le x \le 2$	$-1 \quad min$ $2 \quad max$
89	$y = \frac{x^2 - 2x + 2}{x^2 - x + 1}$	$-1 \le x \le 3$	$0 \quad max$ $2 \quad min$
90	$y = \frac{2 + x}{2 - x} + \frac{2 - x}{2 + x}$	$-1 \le x \le 1$	$\pm 1 \quad max$ $0 \quad min$
91	$y = x^2 - 5x + 7$	$-1 \le x \le 3$	$-1 \quad max$ $\frac{5}{2} \quad min$
92	$y = x^2 + 2x - 3$	$-2 \le x \le 2$	$-1 \quad min$ $2 \quad max$
93	$y = x^4 - 5x^2 + 4$	$-2 \le x \le 2$	$\pm\sqrt{\frac{5}{2}} \quad min$ $0 \quad max$
94	$y = x^4 - 6x^3 - 12x^2 - 8x$	$0 \le x \le 3$	$0 \quad min$ $3 \quad max$

95	$y = \frac{1}{3}x\,(x^2 - 12)$	$-4 \le x \le 4$	$-2 \quad max$ $2 \quad min$
96	$y = x^2 + 3x + 1$	R	$-\frac{3}{2} \quad min$
97	$y = \frac{x^2}{1 + x + x^2}$	R	$-2 \quad max$ $0 \quad min$
98	$y = \frac{3x^2 - 1}{(1 + x^2)^3}$	R	$\pm 1 \quad max$ $0 \quad min$
99	$y = \frac{x^2 + 4}{4x}$	$-3 \le x \le -1$	$-2 \quad max$ $-1 \quad min$
100	$y = \frac{3x^2 + 6}{12x}$	$-5 \le x \le -1$	$-5 \quad min$ $-\sqrt{2} \quad max$
101	$y = \frac{x^2 + 2}{x}$	$-1 \le x \le 1$	$-\sqrt{2} \quad max$ $\sqrt{2} \quad min$
102	$y = \frac{x^3 + x}{x^2 - 1}$	$-2 \le x \le 2$	$-\sqrt{2 + \sqrt{5}} \quad max$ $\sqrt{2 + \sqrt{5}} \quad min$

103	$y = \sqrt[3]{(x+1)^2}$	$-3 \le x \le 4$	-1 min 4 max
104	$y = \sqrt[3]{x^3 - x^2}$	$0 \le x \le 1$	0 max 1 max $\frac{2}{3}$ min
105	$y = \sqrt{x-1} - \sqrt{x+3}$	$1 \le x \le 10$	1 min 10 max
106	$y = \sqrt{8x - x^2}$	$2 \le x \le 6$	2 min 4 max 6 min
107	$y = \frac{1}{\sqrt{x^2 - 7x - 8}}$	$9 \le x \le 11$	9 max 11 min
108	$y = \frac{\sqrt{4x-6}}{2x+3}$	$2 \le x \le 5$	2 min $\frac{9}{2}$ max
109	$y = e^{\frac{x^2+3}{2x}}$	$1 \le x \le 2$	1 max $\sqrt{3}$ min
110	$y = x\, e^{x^2-3x+3}$	$0 \le x \le \frac{11}{10}$	0 min $\frac{1}{2}$ max

111	$y = ln(x^2 - 3x + 2) + \frac{1}{x+2}$	$\frac{5}{2} \leq x \leq 3$	$\frac{5}{2}$ min 3 max
112	$y = ln\frac{x^2+2}{x-3}$	$6 \leq x \leq 8$	$3+\sqrt{11}$ min 8 max
113	$y = (3 + lnx)^2$	R	e^{-3} min
114	$y = ln(x^2 + 2x + 4)$	$-4 \leq x \leq 3$	-1 min 3 max
115	$y = (2x + 1)\, e^{-x}$	$0 \leq x \leq 1$	0 min $\frac{1}{2}$ max
116	$y = ln(x - 1) - ln(x + 3)$	$-6 \leq x \leq -4$	-6 min -4 max
117	$y = \frac{xe^x}{\sqrt{2x-1}}$	$\frac{1}{2} \leq x \leq 2$	$\frac{1}{\sqrt{2}}$ min
118	$y = x\, ln^2 x$	R	1 min

119	$y = x\,e^x$	R	$-1 \quad min$
120	$y = x^2 lnx$	R	$\frac{1}{\sqrt{e}} \quad min$
121	$y = ln(1 - cos\,x)$	$\frac{\pi}{2} \leq x \leq \frac{3}{2}\pi$	$\frac{\pi}{2} \quad min$ $\pi \quad max$ $\frac{3}{2}\pi \quad min$
122	$y = sin\,2x + 2\,cos\,x$	$0 \leq x \leq \pi$	$\frac{\pi}{6} \quad max$ $\frac{5}{6}\pi \quad min$
123	$y = sin^2 x - cos\,x + 1$	$0 \leq x \leq \frac{3}{2}\pi$	$0 \quad min$ $\frac{2}{3}\pi \quad max$
124	$y = x + 2\ cos\,x$	$0 \leq x \leq \frac{\pi}{2}$	$\frac{\pi}{6} \quad max$ $\frac{\pi}{2} \quad min$
125	$y = ln\,sin\,x$	$0 \leq x \leq \frac{\pi}{2}$	$\frac{\pi}{2} \quad max$

126	$y = 8\ sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\ sin^3\left(\frac{x}{2} + \frac{\pi}{4}\right)$	$0 \le x \le \pi$	$\frac{\pi}{6}$ max $\frac{5}{6}\pi$ min
127	$y = 2\|x\| - \|x - 1\|$	R	0 min
128	$y = \left\|\frac{3x - x^3}{1 - 3x^2}\right\|$	R	0 min
129	$y = \frac{x}{\|x^2 + 3x\|}$	R	0 max

3. verificare che i punti indicati sono di massimo o minimo assoluto per le funzioni

130	$y = x^4 + 2x^2 + 1$	± 1 min	no
131	$y = x^2(2x^2 + 1) - 1$	0 max	no
132	$y = x^3 - 6x^2 + 9x + 2$ in $[0;4]$	1 max	si
133	$y = \frac{3x^2 + 4}{2x^2 + 1}$	0 max	si

134	$y = x^3 - \frac{15}{2}x^2 + 18x - 14$	$in \quad \left[1; \frac{7}{2}\right]$	$2 \ min$	no
135	$y = \frac{x}{1 + x + x^2}$		$-1 \ min$	si
136	$y = \frac{x^2 - 4}{(x-1)^2}$	$in \quad [0; 4]$	$4 \ max$	si
137	$y = x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$		$1 \ max$	no
138	$y = \frac{(x+1)^3}{x^2}$	$in \quad [0; 2]$	$2 \ min$	si
139	$y = \sqrt{x - 1}$	$in \quad [1; 6]$	$6 \ max$	si
140	$y = \sqrt{\frac{4 - x^2}{x^2 + 4x - 5}}$		$0 \ min$	no
141	$y = x \, lnx$	$in \quad [1; e]$	$e \ min$	no
142	$y = 2^x x^2$	$in \quad [-3; 0]$	$-\frac{2}{ln\,2} \ max$	si
143	$y = \sqrt{lnx - 3}$		$e^3 \ min$	si

144	$y = 2(e^x - x)$	$in \quad [-2;0]$	$-2 \ min$	no
145	$y = x^2 e^{-x}$	$in \quad [0;2]$	$2 \ max$	si
146	$y = \frac{1}{2}\left[ln(\sqrt{10} - x) + ln(x + \sqrt{10})\right]$	$in \quad [-1;3]$	$3 \ max$	no
147	$y = \frac{e^{\frac{x}{2}}}{x+1}$	$in \quad [0;2]$	$0 \ min$	no
148	$y = cos\, x + sin\, x - 1 - x$	$in \quad \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$	$\frac{\pi}{2} \ min$	si
149	$y = \lvert x + 3 \rvert$		$-3 \ min$	si
150	$y = x^2 - \lvert 7x + 10 \rvert$		$\frac{7}{2} \ max$	no