Identità goniometriche

indice

Gli esercizi sono proposti in ordine di difficoltà crescente.

nota: in un file così lungo e complesso può accadere che sia presente un errore di diversa natura nonostante gli esercizi siano stati controllati più volte. Saremo grati di ricevere segnalazioni di eventuali refusi o suggerimenti di qualsiasi natura.

1. risolubili mediante le relazioni fondamentali

1	$sinx\ cos^2x + sin^3x = sinx$
2	$sin^3x\ cosx + cos^3x\ sinx = sinx\ cosx$
3	$sin^2x - (sinx + cosx)^2 - sinx\ cosx = -cos^2x - 3sinx\ cosx$
4	$sin^4x + 2sin^2x\ cos^2x + cos^4x = 1$
5	$\frac{1}{cos^2x} + \frac{1}{sin^2x} = \frac{1}{cos^2x\ sin^2x}$
6	$cos^4x - sin^4x = 2cos^2x - 1$
7	$\frac{sinx}{1 - cosx} + \frac{sinx}{1 + cosx} = \frac{2}{sinx}$
8	$\frac{cos^3x + sin^3x}{1 + sinx\ cosx} = cosx + sinx$
9	$\frac{1 - sinx}{cosx} + \frac{1 - cosx}{sinx} + \frac{1}{sinx\ cosx} = \frac{sinx + cosx}{sinx\ cosx}$
10	$tan^2x + 1 + \frac{1}{cos^2x} + cos^2x - sin^2x = 2cos^2x - 1$
11	$\frac{1}{cos^2x} - tan^2x + \left(\frac{1 + sinx}{cosx}\right)^2 = \frac{2 + 2sinx}{cos^2x}$
12	$\frac{1}{sin^2x} - 1 - cot^2x + \frac{1}{sinx} - sinx = \frac{cos^2x}{sinx}$

13	$\frac{tanx\ cosx + cotx\ sinx}{sinx\ cosx} = \frac{1}{cosx} + \frac{1}{sinx}$
14	$(1 + cot^2x)(1 - cos^2x) = 1$
15	$\frac{1}{cos^2x} + tan^2x + cot^2x - \left(\frac{1 - cosx}{sinx}\right)^2 - \frac{cosx}{sin^2x} = \frac{cos^4x + cos^3x - 4cos^2x + 2}{cos^2x - cos^4x}$
17	$\frac{1 + cot^2x}{1 + tan^2x} = cot^2x$
18	$cotx + \frac{1}{cotx} = \frac{1}{sinx\ cosx}$
19	$cosx(1 + cosx) = sin^2x\ cot^2x + sinx\ cotx$
20	$sinx + \frac{cos^2x}{sinx} = cscx$
21	$cosx + \frac{sin^2x}{cosx} = secx$
22	$tanx + cotx = secx\ cscx$
23	$tanx - cotx = \frac{1 - 2cos^2x}{sinx\ cosx}$
24	$tanx\ (1 + cotx) = 1 + tanx$
25	$cotx(1 + tanx) = 1 + cotx$

26	$(sinx + cosx)^2 + (sinx - cosx)^2 = 2$
27	$sin^3x = tanx(cosx - cos^3x)$
28	$(1 - cosx)cotx + (cosx - 1)secx\ cscx = sinx - tanx$
29	$\frac{1 + sin\,\alpha\, cos\,\alpha}{sin\,\alpha} = csc\,\alpha + \frac{1}{sec\,\alpha}$
30	$\frac{sin\,\alpha\, cos\,\alpha - 1}{cos\,\alpha} = \frac{1}{csc\,\alpha} - sec\,\alpha$
31	$\frac{(1 - cos\,\alpha)(cos^2\alpha + cos\,\alpha + 1)}{cos\,\alpha} = \frac{2\,sin^2\alpha - sin^4\alpha - 1}{cos^2\alpha} + \frac{1}{cos\,\alpha}$
32	$\frac{cos\,\alpha}{1 - sin^2\alpha} + csc\,\alpha = sin\,\alpha + sec\,\alpha + \frac{1 - sin^2\alpha}{sin\,\alpha}$
33	$\frac{1}{cot\,\alpha} + \frac{sin^2\alpha - 1}{cos^3\alpha} = \frac{1 - sin\,\alpha - cos^2\alpha}{sin\,\alpha\, cos\,\alpha}$
34	$\frac{1}{tan\,\alpha} - \frac{1}{sec\,\alpha} = \frac{cos\,\alpha(1 - sin\,\alpha)}{sin\,\alpha}$
35	$\frac{1}{cos\,\alpha(1 + tan^2\alpha)} + \frac{1}{csc\,\alpha} = cos\,\alpha(tan\,\alpha + 1)$
36	$sin^2\alpha + tan\,\alpha = \frac{sec\,\alpha(1 - cos^2\alpha)}{sin\,\alpha} + \frac{cos^2\alpha - cos^4\alpha}{1 - sin^2\alpha}$
37	$\frac{1}{1 + tan^2\alpha} + \frac{1}{csc\,\alpha} = sin\,\alpha + \frac{1}{sec^2\alpha}$

38	$\cot\alpha + \dfrac{\sin^2\alpha + \sin\alpha - 1}{\sin\alpha\cos\alpha} = \dfrac{\cos\alpha}{1-\sin^2\alpha}$
39	$\sec\alpha - \cos\alpha = \dfrac{\sin^2\alpha}{\cos\alpha}$
40	$\dfrac{2\cos^2\alpha - 1}{1-\sin^2\alpha} = 1 - \tan^2\alpha$
41	$\dfrac{\sin^2\alpha + \tan^2\alpha}{1-\cos^4\alpha} = \sec^2\alpha$
42	$\dfrac{\tan^2\alpha + 1}{\tan^2\alpha - 1} = \dfrac{\tan\alpha + \cot\alpha}{\tan\alpha - \cot\alpha}$
43	$\sin^3\alpha = (\cos\alpha - \cos^3\alpha)\tan\alpha$
44	$\dfrac{\cos^2\alpha - \cos\alpha - 1}{\sin\alpha} + \cot\alpha = \dfrac{\sin^3\alpha - \sin\alpha}{\cos^2\alpha}$
45	$\dfrac{1}{2\cos^2\alpha - 1} = \dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}$
46	$\dfrac{1+\sin\alpha}{\cot\alpha + \cos\alpha} = \dfrac{tg\,\alpha + \sin\alpha}{1+\cos\alpha}$
47	$(\sec\alpha + \tan\alpha)^2 = \dfrac{1+\sin\alpha}{1-\sin\alpha}$
48	$\dfrac{1}{\sin\alpha} + \dfrac{1}{\tan\alpha} = \dfrac{\cos\alpha + 1}{\sin\alpha}$
49	$\dfrac{\cos^2\alpha\,\sin^2\alpha}{\tan^2\alpha} = \cos^4\alpha$

50	$\dfrac{\sin\alpha\cos^2\alpha}{\cot\alpha}=\cos\alpha\,(1-\cos^2\alpha)$
51	$\dfrac{\cos^2\alpha-1}{\tan\alpha}-\dfrac{\sin^2\alpha}{\cot\alpha}=-\tan\alpha$
52	$\dfrac{1+\sin^2\alpha}{\sin\alpha}+\cot\alpha(\cos\alpha-1)=\dfrac{2-\cos\alpha}{\sin\alpha}$
53	$\dfrac{\csc\alpha}{\cos\alpha}-\cot\alpha=\tan\alpha$
54	$\dfrac{\cot\alpha}{\cos\alpha\csc\alpha}=1$
55	$\dfrac{\tan\alpha+\cot\alpha}{\tan\alpha-\cot\alpha}=\dfrac{1}{1-2\cos^2\alpha}$
56	$\dfrac{\cot\alpha}{\sin\alpha}(1+\cos\alpha)+1=\dfrac{1}{1-\cos\alpha}$
57	$\dfrac{\csc\alpha\sin\alpha}{\tan^2\alpha}+1=\dfrac{\cot\alpha}{\sin\alpha\cos\alpha}$
58	$(\sin^2\alpha-\cos^2\alpha)\csc^2\alpha=2-\sec\alpha\dfrac{\cot\alpha}{\sin\alpha}$
59	$\dfrac{1}{\sin\alpha}+\dfrac{\tan\alpha}{\csc\alpha}-1=\dfrac{\cot\alpha}{\cos\alpha}+\sin\alpha(\tan\alpha-\sin\alpha)-\cos^2\alpha$
60	$\dfrac{\sin\alpha+\cos\alpha}{\csc\alpha}=\dfrac{\tan^2\alpha}{1+\tan^2\alpha}+\dfrac{\sin\alpha}{\sec\alpha}$
61	$\dfrac{\sin^2\alpha-\cos^2\alpha}{\sin\alpha+\cos\alpha}=\dfrac{1}{\csc\alpha}-\dfrac{1}{\sec\alpha}$

62	$\frac{\sin\alpha\sec\alpha-1}{1+\tan\alpha}=\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$
63	$\csc\alpha\left(\sin\alpha-\frac{\cot\alpha}{\sec\alpha}\right)=1-\cot^2\alpha$
64	$\frac{\tan^2\alpha-1}{\sec^2\alpha}=1-2\cos^2\alpha$
65	$\frac{\cot\alpha}{\csc\alpha-1}+\frac{\csc\alpha+1}{\cot\alpha}=\frac{2}{\sec\alpha-\tan\alpha}$

2. risolubili mediante angoli associati

66	$\sin^2(\pi-\alpha)+\cos(\pi-\alpha)=1-\cos\alpha-\cos^2\alpha$
67	$\sin^4(\pi-\alpha)=\cos^4\alpha-1+2\sin^2(\pi-\alpha)$
68	$1+\sin\alpha-\sec(\pi-\alpha)\cos(\pi-\alpha)=\sin(\pi-\alpha)\sec\alpha\cos\alpha$
69	$\frac{\cot\alpha}{\cos^2(\pi-\alpha)}=\tan\alpha-\cot(\pi-\alpha)$
70	$\cos\alpha-\sin\alpha=\frac{\cos^2(\pi-\alpha)-\sin^2(\pi-\alpha)}{\sin(\pi-\alpha)-\cos(\pi-\alpha)}$
71	$\sin^2(\pi-\alpha)-\cos(\pi-\alpha)-\sin\alpha\tan(\pi-\alpha)=\sec\alpha+1-\cos^2(\pi-\alpha)$
72	$\cot^2\alpha+\frac{1+2\cos(\pi+\alpha)}{\sin^2(\pi+\alpha)}=\frac{1+\cos(\pi+\alpha)}{1-\cos(\pi+\alpha)}$
73	$-\cos(\pi+\alpha)-\sec\alpha=\sin(\pi+\alpha)\tan(\pi+\alpha)$

74	$1+\tan\alpha=\tan\alpha[1+\cot(\pi+\alpha)]$
75	$\cot(\pi+\alpha)+\tan\alpha=\dfrac{1}{\sin(\pi+\alpha)\cos(\pi+\alpha)}$
76	$\cot^2(\pi+\alpha)=\dfrac{1+\cot^2(\pi+\alpha)}{1+\tan^2(\pi+\alpha)}$
77	$\dfrac{\sec^2(\pi+\alpha)}{\csc(\pi+\alpha)}=[\tan(\pi+\alpha)+\cot\alpha]\dfrac{\sec(\pi+\alpha)}{\csc^2\alpha}$
78	$\csc(2\pi-\alpha)=-\cos(2\pi-\alpha)\cot\alpha-\sin\alpha$
79	$\sin(2\pi-\alpha)\cot(2\pi-\alpha)+1=-\dfrac{\sin\alpha+\tan\alpha}{\tan(2\pi-\alpha)}$
80	$\dfrac{\cos(2\pi-\alpha)}{1+\sin(2\pi-\alpha)}=\dfrac{1-\sin(2\pi-\alpha)}{\cos(2\pi-\alpha)}$
81	$\sin(2\pi-\alpha)\tan(2\pi-\alpha)=\sec(2\pi-\alpha)-\cos(2\pi-\alpha)$
82	$\dfrac{\sin(2\pi-\alpha)}{\sec^2(2\pi-\alpha)}=\dfrac{\cos(2\pi-\alpha)}{\cot(2\pi-\alpha)+\tan(2\pi-\alpha)}$
83	$-\cot\alpha=\dfrac{\sin(2\pi-\alpha)}{1-\cos(-\alpha)}-\csc(2\pi-\alpha)$
84	$\dfrac{1+\tan^2(-\alpha)}{1-\tan^2(2\pi-\alpha)}=\dfrac{1}{2\cos^2(-\alpha)-1}$
85	$\dfrac{\cos(2\pi-\alpha)}{1-\sin(-\alpha)}=\tan(-\alpha)+\sec(2\pi-\alpha)$

87	$\frac{\cos(\pi-\alpha)}{\sin(\pi+\alpha)}=\cot\alpha$
88	$\frac{\tan(2\pi-\alpha)}{\sin(\pi-\alpha)}-\cos(\pi+\alpha)=-\frac{\tan\alpha}{\csc\alpha}$
89	$\frac{\cot(2\pi+\alpha)\tan(\pi+\alpha)}{\sin(2\pi-\alpha)\cos(\pi+\alpha)}=\sec\alpha\csc\alpha$
90	$\frac{\sin(\pi+\alpha)-\sin\alpha}{\csc\alpha-\csc(\pi+\alpha)}=\cos^2\alpha-1$
91	$\frac{\sin\alpha+\tan(\pi-\alpha)}{\cot(\pi-\alpha)+\cos(\pi+\alpha)}=\tan^2\alpha\frac{1-\cos\alpha}{1+\sin\alpha}$
92	$\frac{\sin(2\pi+\alpha)\cot(\pi+\alpha)}{\cos(\pi-\alpha)}=\cos(2\pi-\alpha)\tan(2\pi+\alpha)\csc(\pi+\alpha)$
93	$\csc(\pi+\alpha)-\sec(2\pi-\alpha)=\csc(\pi-\alpha)\sec(\alpha)(\sin(\pi+\alpha)+\cos(\pi-\alpha))$
94	$\frac{\tan(2\pi-\alpha)-\sec(\pi+\alpha)}{\tan(3\pi+\alpha)-\sec(\alpha-2\pi)}=-\frac{1+\cot^2\alpha}{\csc^2\alpha}$
95	$\frac{\cos(3\pi-\alpha)-\sec(\alpha-2\pi)}{\tan(7\pi+\alpha)-\cot(-\pi-\alpha)}=\sin(3\pi-\alpha)(\sin^2\alpha-2)$
96	$\frac{\cos^3\left(\frac{\pi}{2}-\alpha\right)-\sin^3\left(\frac{\pi}{2}+\alpha\right)}{\sin(\pi-\alpha)-\sin\left(\frac{\pi}{2}-\alpha\right)}=\sin\alpha(\cos(2\pi-\alpha)+\csc\alpha)$
97	$\frac{\cos^2(\pi-\alpha)-\cos^2\left(\frac{\pi}{2}+\alpha\right)}{\sin\left(\frac{\pi}{2}+\alpha\right)+\cos\left(\frac{\pi}{2}+\alpha\right)}=\frac{1+\tan(\pi+\alpha)}{\sec(4\pi-\alpha)}$

98	$\left(1+tan^2(3\pi-\alpha)\right)\left(sin^2\left(\frac{\pi}{2}+\alpha\right)-cos^2\left(3\frac{\pi}{2}-\alpha\right)\right)=2-sec^2(\pi+\alpha)$
99	$\frac{a^2\,cos(2\pi-\alpha)-b^2\,sin\left(\frac{\pi}{2}+\alpha\right)}{a\,tan(\pi-\alpha)+b\,cot\left(\frac{\pi}{2}-\alpha\right)}=(a+b)\left(cos\left(\frac{\pi}{2}-\alpha\right)-csc\,\alpha\right),\qquad a\neq b$
100	$\frac{b^3\,cos(\pi-\alpha)+a^3 sin\left(3\frac{\pi}{2}-\alpha\right)}{a^2 cot\,(\pi+\alpha)-b^2 tan\left(\frac{\pi}{2}+\alpha\right)+ab\,cot(3\pi-\alpha)}=\frac{a+b}{sec\left(3\frac{\pi}{2}-\alpha\right)}$
101	$\frac{(a+b)^2 tan(2\pi-\alpha)+4ab\,cot\left(\frac{\pi}{2}-\alpha\right)}{a\,tan\left(\frac{\pi}{2}-\alpha\right)+b\,tan\left(\frac{\pi}{2}+\alpha\right)}=(b-a)\frac{sin^2(\pi-\alpha)}{1-cos^2\left(\frac{\pi}{2}+\alpha\right)}$
102	$\frac{a^4 sec\left(\frac{3\pi}{2}+\alpha\right)+b^4\,sec\left(\frac{3\pi}{2}-\alpha\right)}{(a-b)^2\,sec(\pi-\alpha)-2ab\,csc\,\left(\frac{\pi}{2}+\alpha\right)}=(a^2-b^2)tan\left(\frac{\pi}{2}+\alpha\right)$

3. risolubili mediante formule goniometriche

103	$sin(\alpha+\beta)\,sin(\alpha-\beta)=sin^2\,\alpha-sin^2\,\beta$
104	$sin\,\alpha\,cos(\alpha+\beta)=cos\,\alpha\,sin(\alpha+\beta)-sin\,\beta$
105	$sin\,\alpha\,sin(\alpha-\beta)+cos\,\alpha\,cos(\alpha-\beta)=cos\,\beta$
106	$\frac{cos(\alpha+\beta)+cos(\alpha-\beta)}{sin(\alpha+\beta)+sein(\alpha-\beta)}=cot\,\alpha$
107	$\frac{cos(\alpha-\beta)}{cos(\alpha+\beta)}=\frac{1+tan\,\alpha\,tan\,\beta}{1-tan\,\alpha\,tan\,\beta}$
108	$cos(\alpha-\beta)=(tan\,\alpha+cot\,\beta)\,sin\,\beta\,cos\,\alpha$

110	$(tan\,\alpha + tan\,\beta)[cos(\alpha+\beta) + cos(\alpha-\beta)] = 2\,sin(\alpha+\beta)$
111	$sin\,11\alpha - sin\,\alpha = 2\,cos\,6\alpha\,sin\,5\alpha$
112	$\frac{cos\,4\alpha - cos\,8\alpha}{cos\,4\alpha + cos\,8\alpha} = tan\,6\alpha\,tan\,2\alpha$
113	$2\,cos\,2\alpha\,cos\,3\alpha = cos\,5\alpha + cos\,\alpha$
114	$\frac{sin\,3\alpha + sin\,\alpha}{sin\,5\alpha - sin\,\alpha} = cos\,\alpha\,sec\,3\alpha$
115	$2\,sin\,3\alpha\,sin\,8\alpha = cos\,5\alpha - cos\,11\alpha$
116	$cos^4\alpha - sin^4\alpha = cos\,2\alpha$
117	$2\,cos\,2\alpha = (1 - cos\,2\alpha)(ctg^2\alpha - 1)$
118	$2\,sin\,2\alpha\,cos\,\alpha - sin\,3\alpha = sin\,\alpha$
119	$\frac{sin\,3\alpha - sin\,\alpha}{cos\,\alpha - cos\,3\alpha} = cot\,2\alpha$
120	$2\,sin^2\frac{\alpha}{2}\,tan\,\alpha = tan\,\alpha - sin\,\alpha$
121	$2\,cos^2\frac{\alpha}{2} - cos\,\alpha = 1$
122	$2\,cos\,\alpha = (1 - cos\,\alpha)\left(cot^2\frac{\alpha}{2} - 1\right)$

123	$cos(\alpha+\beta)-cos(\alpha-\beta)=-2sin\,\alpha\,sin\,\beta$
124	$sin(\alpha+\beta)+sin(\alpha-\beta)=2sin\,\alpha\,cos\,\beta$
125	$sin(\alpha-\beta)-cos(\alpha+\beta)=(sin\,\alpha-cos\,\alpha)(cos\,\beta+sin\,\beta)$
126	$sin^2(\alpha+\beta)-cos^2(\alpha-\beta)=(1-2sin^2\beta)(2sin^2\alpha-1)$
127	$\dfrac{cos(\alpha+\beta)}{sin(\alpha-\beta)+sin(\alpha+\beta)}=\dfrac{1}{2}(cot\,\alpha-tan\,\beta)$
128	$sin(\alpha+\beta)sin(\alpha-\beta)=sin^2\alpha-sin^2\beta$
129	$\dfrac{cos(\alpha-\beta)}{csc(\alpha+\beta)}=sin\,\alpha\,cos\,\alpha+sin\,\beta\,cos\,\beta$
130	$cos\,2\alpha-sin\,2\alpha=(cos\,\alpha-sin\,\alpha)^2-2sin^2\alpha$
131	$sin\,2\alpha=2\dfrac{tan\,\alpha}{sec^2\alpha}$
132	$cos\,\alpha\,sin\,2\alpha-cos\,2\alpha\,sin\,\alpha=sin\,\alpha$
134	$\dfrac{cos(\alpha+\beta)+cos(\alpha-\beta)}{2}=cos\,\alpha\,cos\,\beta$
135	$\dfrac{sin\,2\alpha\,sec\,\alpha\,csc\,\alpha}{2}=1$
136	$sec\,2\alpha\,cot\,2\alpha+csc\,2\alpha\,sin^2\alpha=sec\,\alpha\,csc\,\alpha-\dfrac{1}{2}cot\,\alpha$

137	$1-cos^2 2\alpha = 4tan^2\alpha\, cos^4\alpha$
138	$\frac{1}{2}tan\,2\alpha + cot\,\alpha = \frac{cot\,\alpha\, cos^2\alpha}{cos\,2\alpha}$
139	$cos^2\alpha + cos\,2\alpha = cos^2\alpha\,(2-tan^2\alpha)$

4. di riepilogo

140	$\frac{1}{csc\,\alpha} + \frac{1-sin\,\alpha\, cos\,\alpha}{cos\,\alpha} = \frac{tan\,\alpha}{sin\,\alpha}$
141	$sin\,\alpha\, cos\,\alpha\, csc\,\alpha + sec\,\alpha = \frac{2-sin^2\alpha}{cos\,\alpha}$
142	$tan\,\alpha + sin^2\alpha = \frac{sec\,\alpha(1-cos^2\alpha)}{sin\,\alpha} + \frac{cos^2\alpha - cos^4\alpha}{1-sin^2\alpha}$
143	$\frac{1}{cot\,\alpha} - \frac{1}{tan\,\alpha} = \frac{2\,sin^2\alpha - 1}{sin\,\alpha\, cos\,\alpha}$
144	$sin\,\alpha = \frac{tan\,\alpha}{\pm\sqrt{1+tan^2\alpha}}$
145	$cos\,\alpha = \pm\frac{\sqrt{csc^2\alpha - 1}}{csc\,\alpha}$
146	$\frac{2\,sin^2(\pi-\alpha)-1}{sin\,\alpha\, cos(2\pi-\alpha)} + cos(-\alpha)\,csc(\pi-\alpha) = \frac{1}{cot(\pi+\alpha)}$
147	$1+2\,sin(2\pi-\alpha)\,cos(\pi+\alpha) = [sin(\pi-\alpha) - cos(\pi+\alpha)]^2$
148	$\frac{1+cos(\pi+\alpha)}{sin(2\pi-\alpha)\,cos\,\alpha} = sin\,\alpha - tan\,\alpha - \frac{cos(\pi+\alpha)+1}{tan(\pi+\alpha)}$

149	$[sin(\pi-\alpha)-cos(\pi-\alpha)]^2=\frac{2\,sin(\pi-\alpha)}{sec(2\pi-\alpha)}+1$
150	$\frac{tan(\pi+\alpha)}{1-tan^2(\pi-\alpha)}=\frac{cot(\pi+\alpha)}{cot^2(2\pi-\alpha)-1}$
151	$-sin^3(2\pi-\alpha)=tan(\pi+\alpha)[cos(-\alpha)+cos^3(\pi-\alpha)]$
152	$\frac{sin\,\alpha(1+sin\,\alpha)\,tan(2\pi-\alpha)}{tan\,\alpha}=sin(\pi+\alpha)\,[1-sin(2\pi-\alpha)]$
153	$\frac{1}{2}sin\,2(\alpha+\beta)=sin(\alpha+\beta)\,cos(\alpha+\beta)$
154	$2\,sin(\alpha-\beta)\,cos(\alpha+\beta)=sin\,2\alpha-sin\,2\beta$
155	$cos^4\frac{\alpha}{2}-sin^4\frac{\alpha}{2}=cos\,\alpha$
156	$csc\,\alpha+cot\,\alpha=cot\frac{\alpha}{2}$
157	$\frac{2\,sin\,\alpha-sin\,2\alpha}{2\,sin\,\alpha+sin\,2\alpha}=tan^2\frac{\alpha}{2}$
158	$\frac{sin(3\alpha+\beta)\,sin(3\alpha-\beta)-sin(\alpha+\beta)\,sin(\alpha-\beta)}{sin\,4\alpha\,sin\,2\alpha}=1$
159	$\frac{cos\,\beta}{sin\,\beta-cos\,\alpha}-\frac{cos\,\beta}{sin\,\beta+cos\,\alpha}=\frac{cos\,\alpha}{sin\,\alpha-cos\,\beta}-\frac{cos\,\alpha}{sin\,\alpha+cos\,\beta}$
160	$tan^2\alpha-sin^2\alpha=sin^2\alpha\,tan^2\alpha$
161	$cos\,3\alpha\;\;sec\,\alpha=4cos^2\alpha-3$

162	$\sin 3\alpha = \sin^3\alpha(3\csc^2\alpha - 4)$
163	$\dfrac{\sec(\pi-\alpha)}{2\sin(-\alpha)} = \csc 2\alpha$
164	$\dfrac{2\sin\left(\alpha+\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \sqrt{3}\tan(3\pi+\alpha) + \sec\left(\frac{\pi}{2}+\alpha\right)\sin(\alpha-\pi)$
165	$\sqrt{2}\dfrac{\sin\frac{3}{4}\pi}{\tan(\pi+2\alpha)} = -\cot(-2\alpha)$
166	$\sqrt{2}\dfrac{\sin\left(\alpha+\frac{\pi}{4}\right)}{\cos(\alpha-2\pi)} = 1 - \tan(-\alpha-3\pi)$
167	$\tan\left(\alpha+\frac{\pi}{4}\right)\left(\cos(-\alpha)+\sin(-\alpha)\right) = \sqrt{2}\cos\left(\alpha-\frac{\pi}{4}\right)$
168	$\sin^4\alpha + \cos^4\alpha + \frac{1}{2}\sin^2 2\alpha = 1$
169	$\dfrac{1}{\sin(\pi-\alpha)+1} - \dfrac{1}{\sin(\pi+\alpha)+1} = \dfrac{\sin 2\alpha}{\cos^3(3\pi+\alpha)}$
170	$\sec^2\left(\frac{\pi}{2}+\alpha\right)\csc^2\left(\alpha-\frac{\pi}{2}\right) = \dfrac{1+\cot^2\left(\frac{\pi}{2}+\alpha\right)}{1-\cos^2(\pi+\alpha)}$
171	$\left(\cos(2\pi-\alpha) - \sin(\pi+\alpha)\right)^2 - \dfrac{2}{1+\cot^2\alpha} = \cos 2\alpha + \sin 2\alpha$

172	$\dfrac{\cos^2\alpha - \cos^2\left(\frac{3}{2}\pi + \alpha\right)}{1 + \tan(7\pi + \alpha)} = \dfrac{1 + \cos 2\alpha - \sin 2\alpha}{2}$
173	$\cos\left(\alpha - \frac{\pi}{3}\right) + \cos\left(\alpha - \frac{2}{3}\pi\right) = \tan\frac{\pi}{3}\sin\alpha$
174	$\dfrac{\sqrt{2}\sin\left(2\alpha + \frac{\pi}{4}\right) + 1}{2\left(\cos\left(\frac{\pi}{2} - \alpha\right) + \sin\left(\frac{\pi}{2} + \alpha\right)\right)} = \cos\alpha$
175	$\dfrac{1 + \tan(-\alpha)}{-1 + \tan(\pi - \alpha)} = \dfrac{\cos\left(\alpha + \frac{\pi}{4}\right)}{\cos\left(\alpha + 3\frac{\pi}{4}\right)}$
176	$\dfrac{\cos\left(\alpha - \frac{\pi}{3}\right) + \sin\left(\alpha + \frac{\pi}{3}\right)}{\sqrt{3} + 1} = \dfrac{\sqrt{2}}{2}\sin\left(\alpha + \frac{\pi}{4}\right)$
177	$\dfrac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2sen(\alpha + \beta)} = [\tan(\alpha + \beta)(1 - \tan\alpha\tan\beta)]^{-1}$