

Diseguazioni goniometriche intere

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Gli esercizi sono proposti in ordine di difficoltà crescente.

nota: in un file così lungo e complesso può accadere che sia presente un errore di diversa natura nonostante gli esercizi siano stati controllati più volte. Saremo grati di ricevere segnalazioni di eventuali refusi o suggerimenti di qualsiasi natura.

1. disequazioni elementari



1	$\sin x > 0$	$2k\pi < x < \pi + 2k\pi$
2	$\sin x < 0$	$-\pi + 2k\pi < x < 2k\pi$
3	$\cos x > 0$	$-\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$
4	$\cos x < 0$	$\frac{\pi}{2} + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$
5	$\sin x > -1$	$x \neq \frac{3}{2}\pi + 2k\pi$
6	$\sin x < 1$	$x \neq \frac{\pi}{2} + 2k\pi$
7	$\cos x > 1$	\emptyset
8	$\cos x \leq -1$	$x = \pi + 2k\pi$
9	$\sin x \geq 1$	$x = \frac{\pi}{2} + 2k\pi$
10	$\sin x < -1$	\emptyset
11	$\cos x \leq 1$	\mathbb{R}
12	$\cos x > -1$	$x \neq \pi + 2k\pi$

13	$\sin x > \frac{1}{2}$	$\frac{\pi}{6} + 2k\pi < x < \frac{5}{6}\pi + 2k\pi$
14	$\cos x > -\frac{1}{2}$	$-\frac{2}{3}\pi + 2k\pi < x < \frac{2}{3}\pi + 2k\pi$
15	$\cos x > \frac{\sqrt{3}}{2}$	$-\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{6} + 2k\pi$
16	$\sin x < -\frac{\sqrt{2}}{2}$	$\frac{5}{4}\pi + 2k\pi < x < \frac{7}{4}\pi + 2k\pi$
17	$\sin x \leq \frac{\sqrt{3}}{2}$	$-\frac{4}{3}\pi + 2k\pi \leq x \leq \frac{\pi}{3} + 2k\pi$
18	$\cos x > \frac{\sqrt{2}}{2}$	$-\frac{\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi$
19	$\cos x \leq \frac{\sqrt{3}}{2}$	$\frac{\pi}{6} + 2k\pi < x < \frac{11}{6}\pi + 2k\pi$
20	$\sin x > \frac{\sqrt{2}}{2}$	$\frac{\pi}{4} + 2k\pi < x < \frac{3}{4}\pi + 2k\pi$
21	$\tan x > 0$	$k\pi < x < \frac{\pi}{2} + k\pi$
22	$\cot x < 0$	$-\frac{\pi}{2} + k\pi < x < k\pi$
23	$\tan x < 0$	$\frac{\pi}{2} + k\pi < x < \pi + k\pi$
24	$\cot x \geq 0$	$k\pi \leq x \leq \frac{\pi}{2} + k\pi$

25	$\cot x < -\sqrt{3}$	$\frac{5}{6}\pi + k\pi < x < \pi + k\pi$
26	$\tan x - \sqrt{3} < 0$	$-\frac{\pi}{2} + k\pi < x < \frac{\pi}{3} + k\pi$
27	$\cot x + \sqrt{3} > 0$	$k\pi < x < \frac{5}{6}\pi + 2k\pi$
28	$\cot x < -1$	$\frac{3}{4}\pi + k\pi < x < \pi + k\pi$
29	$\tan x > \sqrt{3}$	$\frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi$
30	$\tan x > -1$	$-\frac{\pi}{4} + k\pi < x < \frac{\pi}{2} + k\pi$
31	$\tan x < \frac{\sqrt{3}}{3}$	$-\frac{\pi}{2} + k\pi < x < \frac{\pi}{6} + k\pi$
32	$2\sin x + \sqrt{2} > 0$	$-\frac{\pi}{4} + 2k\pi < x < \frac{5}{4}\pi + 2k\pi$
33	$2\cos x + \sqrt{2} > 0$	$-\frac{3}{4}\pi + 2k\pi < x < \frac{3}{4}\pi + 2k\pi$
34	$\tan x < -\sqrt{3}$	$-\frac{\pi}{2} + k\pi < x < -\frac{\pi}{3} + k\pi$
35	$\tan x < 2 + \sqrt{3}$	$-\frac{\pi}{2} + k\pi < x < \frac{5}{12}\pi + k\pi$
36	$3\sin x - 10 > 0$	\emptyset

37	$2\cos x + \sqrt{3} \geq 0$	$-\frac{5}{6}\pi + 2k\pi \leq x \leq \frac{5}{6}\pi + 2k\pi$
38	$\cot x < -\frac{\sqrt{3}}{3}$	$-\frac{\pi}{3} + k\pi < x < k\pi$
39	$\tan 2x - 1 < 0$	$-\frac{\pi}{4} + k\frac{\pi}{2} < x < \frac{\pi}{8} + k\frac{\pi}{2}$
40	$\sin\left(x + \frac{\pi}{4}\right) > 0$	$-\frac{\pi}{4} + 2k\pi < x < \frac{3}{4}\pi + 2k\pi$
41	$2\cos\left(x - \frac{\pi}{3}\right) < 1$	$\frac{2}{3}\pi + 2k\pi < x < \frac{7}{4}\pi + 2k\pi$
42	$\cos\left(2x - \frac{\pi}{4}\right) < \frac{\sqrt{2}}{2}$	$\frac{\pi}{4} + k\pi < x < \pi + k\pi$
43	$\cot x < \sqrt{5 + 2\sqrt{5}}$	$\frac{\pi}{10} + k\pi < x < \pi + k\pi$
44	$\tan x - 2 - \sqrt{3} < 0$	$-\frac{\pi}{2} + k\pi < x < \frac{5}{12}\pi + k\pi$
45	$\cos x < \frac{\sqrt{6} + \sqrt{2}}{4}$	$\frac{\pi}{12} + 2k\pi < x < \frac{23}{12}\pi + 2k\pi$


2. disequazioni di secondo grado



46	$3\cos^2 x + 1 \leq 0$	\emptyset
47	$\sqrt{3}\sin x - 2\sin^2 x \geq 0$	$2k\pi \leq x \leq \frac{\pi}{3} + 2k\pi \vee$ $\frac{2}{3}\pi + 2k\pi \leq x \leq \pi + 2k\pi$

48	$4\sin^2 x - 1 > 0$	$\frac{\pi}{6} + k\pi < x < \frac{5}{6}\pi + k\pi$
49	$\cos^2 x + 2\cos x \leq 0$	$\frac{\pi}{2} + 2k\pi \leq x \leq \frac{3}{2}\pi + 2k\pi$
50	$2\sin^2 x - \sin x - 1 \leq 0$	$-\frac{\pi}{6} + 2k\pi \leq x \leq \frac{7}{6}\pi + 2k\pi$
51	$\sqrt{2}\sin^2 x + \sin x < 0$	$\pi + 2k\pi < x < \frac{5}{4}\pi + 2k\pi \vee$ $\frac{7}{4}\pi + 2k\pi < x < \pi + 2k\pi$
52	$2\cos^2 x + 3\cos x - 2 < 0$	$\frac{\pi}{3} + 2k\pi < x < \frac{5}{3}\pi + 2k\pi$
53	$\tan^2 x - \tan x > 0$	$\frac{\pi}{4} + k\pi < x < \pi + k\pi \wedge$ $x \neq \frac{\pi}{2} + k\pi$
54	$\tan^2 x - (1 + \sqrt{3})\tan x + \sqrt{3} > 0$	$-\frac{\pi}{2} + k\pi < x < \frac{\pi}{4} + k\pi \vee$ $\frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi$
55	$\sin^2 x + 3\cos x - 3 > 0$	\emptyset
56	$\cos 2x - 2\cos x + 1 > 0$	$\frac{\pi}{2} + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$
57	$2\cos^2 \frac{x}{2} < 1 - 2\cos x$	$\frac{\pi}{2} + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$

58	$2\sin^2 \frac{x}{2} \leq 1 - 2\sin x$	$-\frac{3}{4}\pi + 2k\pi \leq x \leq \frac{\pi}{4} + 2k\pi$
59	$2\sin^2 x + 5\cos x - 4 > 0$	$-\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi$
60	$2\sin^2 x - \sqrt{2}\sin x > 0$	$-\pi + 2k\pi < x < 2k\pi \vee$ $\frac{\pi}{4} + 2k\pi < x < \frac{3}{4}\pi + 2k\pi$
61	$3\tan^2 x - 1 > 0$	$-\frac{\pi}{6} + k\pi < x < -\frac{\pi}{2} + k\pi \vee$ $-\frac{\pi}{2} + k\pi < x < -\frac{5}{6}\pi + k\pi$
62	$2\cos^2 x - \sqrt{2}\cos x > 0$	$\frac{\pi}{2} + 2k\pi < x < \frac{3}{2}\pi + 2k\pi \vee$ $-\frac{\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi$
63	$8\cos^2 x + 2\cos x - 3 < 0$	$-\arccos\left(-\frac{3}{4}\right) + 2k\pi < x < -\frac{\pi}{3} + 2k\pi \vee$ $\frac{\pi}{3} + 2k\pi < x < \arccos\left(-\frac{3}{4}\right) + 2k\pi$
64	$\tan^2 x + 2\tan x + 3 < 0$	<i>impossibile</i>
65	$2\cos^2 x - \sin x - 1 < 0$	$\frac{\pi}{6} + 2k\pi < x < \frac{5}{6}\pi + 2k\pi$
66	$3\cot^2 x - 4\sqrt{3}\cot x + 3 > 0$	$-\frac{2}{3}\pi + k\pi < x < 2\pi + k\pi \vee$ $k\pi < x < \frac{\pi}{6} + k\pi$
67	$\tan^2 x + (\sqrt{3} + 1)\tan x + \sqrt{3} > 0$	$x \neq k\pi \vee x \neq \pm \frac{\pi}{2}$

68	$2\cos^2 x + \sqrt{3}\sin x - 2 > 0$	$2k\pi < x < \frac{\pi}{3} + 2k\pi \vee$ $\frac{2}{3}\pi + 2k\pi < x < (2k + 1)\pi$
69	$\cos 2x + \sin x \geq 0$	$-\pi + 2k\pi < x < \frac{1}{6}(12k\pi - 5\pi) \vee$ $-\frac{\pi}{6} + 2k\pi \leq x \leq \pi + 2k\pi$
3. disequazioni lineari in seno e coseno 		
70	$\sin x + \cos x \leq 0$	$\frac{3}{4}\pi + 2k\pi \leq x \leq \frac{7}{4}\pi + 2k\pi$
71	$\sin x - \cos x > 0$	$\frac{\pi}{4} + 2k\pi < x < \frac{5}{4}\pi + 2k\pi$
72	$\sin x - \cos x \leq 0$	$-\frac{3}{4}\pi + 2k\pi \leq x \leq \frac{\pi}{4} + 2k\pi$
73	$\sin x + \sqrt{3}\cos x > 0$	$-\frac{\pi}{3} + 2k\pi < x < \frac{2}{3}\pi + 2k\pi$
74	$3\sin x - \sqrt{3}\cos x > 0$	$-\frac{5}{6}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi$
75	$\sqrt{3}\sin x + 3\cos x > 0$	$-\frac{\pi}{3} + 2k\pi < x < \frac{2}{3}\pi + 2k\pi$
76	$\sqrt{3}\sin x + 3\cos x - \sqrt{3} > 0$	$-\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$
77	$\sqrt{3}\sin x - \cos x - 1 < 0$	$-\pi + 2k\pi < x < \frac{\pi}{3} + 2k\pi$

78	$\cos x + \sqrt{3}\sin x - \sqrt{3} > 0$	$\frac{\pi}{6} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$
79	$\cos x - \sin x + 1 > 0$	$-\pi + 2k\pi < x < \frac{\pi}{2} + 2k\pi$
80	$\cos x + \sin x - \sqrt{2} \geq 0$	$x = \frac{\pi}{4} + 2k\pi$
81	$\cos x + \sqrt{3}\sin x - \sqrt{3} \geq 0$	$\frac{\pi}{6} + 2k\pi \leq x \leq \frac{\pi}{2} + 2k\pi$
82	$\sin\left(x - \frac{\pi}{3}\right) + \cos\left(\frac{4}{3}\pi - x\right) - 1 < 0$	$-\frac{2}{3}\pi + 2k\pi < x < \frac{5}{6}\pi + 2k\pi$
83	$\sin 2x - \cos x + 1 > 2\sin x$	$2k\pi < x < \frac{\pi}{6} + 2k\pi \vee$ $\frac{5}{6}\pi + 2k\pi < x < 2k\pi$
84	$2\cos x - 2\sin\left(-x - \frac{2}{3}\pi\right) - 1 < 0$	$-\pi + 2k\pi < x < \frac{\pi}{2} + 2k\pi \vee$ $\frac{\pi}{3} + 2k\pi < x < \pi + 2k\pi$

4. disequazioni omogenee di secondo grado in seno e coseno



85	$\sin^2 x - 3\cos^2 x \leq 0$	$-\frac{\pi}{3} + k\pi \leq x \leq \frac{\pi}{3} + k\pi$
86	$\cos^2 x + (\sqrt{3} - 1)\sin x \cos x - \sqrt{3}\sin^2 x > 0$	$-\frac{\pi}{6} + k\pi < x < \frac{\pi}{4} + k\pi$
87	$3\sin^2 x - 2\sin x \cos x - \cos^2 x < 0$	$-\arctan \frac{1}{3} + k\pi < x < \frac{\pi}{4} + k\pi$

88	$\sin^2 x - (\sqrt{3} + 1)\sin x \cos x + \sqrt{3}\cos^2 x > 0$	$-\frac{2}{3}\pi + k\pi < x < \frac{\pi}{4} + k\pi$
89	$\sin^2 x + 4\sin x \cos x + 3\cos^2 x > 0$	$-\frac{\pi}{4} + k\pi < x < \pi - \arctan 3 + k\pi$
90	$2\sqrt{3}\cos^2 x - 2\sin x \cos x - \sqrt{3} \leq 0$	$\frac{\pi}{6} + k\pi < x < \frac{2}{3}\pi + k\pi$
91	$3\cos^2 x + 2\sin 2x + 2\sin^2 x > 2$	$-\arctan \frac{1}{4} + k\pi < x < \frac{\pi}{2} + k\pi$
92	$\sin^2 x + 4\sin x \cos x + \cos^2 x < 0$	$\frac{\pi}{12} + k\pi < x < \frac{5}{12}\pi + k\pi$
93	$5\sin^2 x - \sqrt{3}\sin 2x - \cos^2 x < 2$	$-\frac{\pi}{6} + k\pi < x < \frac{\pi}{3} + k\pi$
94	$3\sin^2 x + \sqrt{3}\sin 2x + \cos^2 x > 0$	$R - \left\{ \frac{5}{6}\pi + k\pi \right\}$
95	$(3 + \sqrt{3})\sin^2 x + (\sqrt{3} - 1)\sin x \cos x + 2\cos^2 x > 3$	$\frac{\pi}{6} + k\pi < x < \frac{3}{4}\pi + k\pi$

5. disequazioni di riepilogo



96	$3 \tan x > \sqrt{3}$	$-\frac{5}{6}\pi + k\pi < x < -\frac{\pi}{2} + k\pi$
97	$\tan x \geq -\sqrt{3}$	$-\frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi$

98	$2 \sin x - \sqrt{3} \geq 0$	$\frac{\pi}{3} + 2k\pi \leq x \leq \frac{2}{3}\pi + 2k\pi$
99	$\cos x - 2 \geq 0$	<i>impossibile</i>
100	$2 \sin x + \sqrt{3} > 0$	$-\frac{\pi}{3} + 2k\pi < x < \frac{4}{3}\pi + 2k\pi$
101	$2 \sin^2 x - (2 - \sqrt{3}) \sin x - \sqrt{3} \leq 0$	$-\frac{\pi}{3} + 2k\pi \leq x \leq \frac{4}{3}\pi + 2k\pi$
102	$\cos^2 x + \cos x \geq 0$	$-\frac{\pi}{2} + 2k\pi \leq x \leq \frac{\pi}{2} + 2k\pi \vee$ $x = \pi + 2k\pi$
103	$\cos^2 x + 1 \leq 0$	<i>impossibile</i>
104	$2 \cos^2 x + 3 \cos x + 1 > 0$	$-\frac{2}{3}\pi + 2k\pi < x < \frac{2}{3}\pi + 2k\pi$
105	$2 \sin^2 x - \sin x - 1 > 0$	$-\frac{5}{6}\pi + 2k\pi < x < -\frac{\pi}{6} + 2k\pi$
106	$2 \sin^2 x - 3 \sin x + 1 < 0$	$\frac{\pi}{6} + 2k\pi < x < \frac{5}{6}\pi + 2k\pi \wedge$ $x \neq \frac{\pi}{2}$
107	$2 \cos^2 x - 3 \cos x + 1 < 0$	$-\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi \wedge$ $x \neq 0$

108	$\cos 2x + \cos x < 0$	$\frac{\pi}{3} + 2k\pi < x < \frac{5}{3}\pi + 2k\pi \wedge$ $x \neq \pi$
109	$\sin x \cos x > 0$	$k\pi < x < \frac{\pi}{2} + k\pi$
110	$4 \sin^2 x - 2\sqrt{3} \sin x \cos x - 2 \cos^2 x - 1 > 0$	$\frac{\pi}{3} + k\pi < x < \frac{5}{6}\pi + k\pi \wedge$ $x \neq \frac{\pi}{2} + k\pi$
111	$\sin^2 x + \sin x \cos x < 0$	$-\frac{\pi}{4} + k\pi < x < k\pi$
112	$\sqrt{3} \sin x - \cos x \leq 0$	$-\frac{5}{6}\pi + 2k\pi \leq x \leq \frac{\pi}{6} + 2k\pi$
113	$\sqrt{3} \cos x - \sin x + 1 \geq 0$	$-\frac{5}{6}\pi + 2k\pi \leq x \leq \frac{\pi}{2} + 2k\pi$
114	$\sin^4 x - \cos^4 x < 0$	$-\frac{\pi}{4} + k\pi < x < \frac{\pi}{4} + k\pi$
115	$2 \sin^2 x + 4 \cos^2 x > 5 \cos x$	$\frac{\pi}{3} + 2k\pi < x < \frac{5}{3} + 2k\pi$
116	$\cos^2 x + 2 \cos x < 0$	$\frac{\pi}{2} + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$